TABLE OF INTEGRALS

Substitution Rule	**Integration by Parts**
$\int f(g(x))g'(x)\,dx = \int f(u)\,du \quad (u = g(x))$	$\int u\,dv = uv - \int v\,du$
$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$	$\int_a^b uv'\,dx = uv\Big\vert_a^b - \int_a^b vu'\,dx$

Basic Integrals

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C;\ n \neq -1$

2. $\int \frac{dx}{x} = \ln|x| + C$

3. $\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

4. $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

5. $\int \tan x\,dx = \ln|\sec x| + C$

6. $\int \cot x\,dx = \ln|\sin x| + C$

7. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

8. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

9. $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

10. $\int b^{ax}\,dx = \frac{1}{a\ln b}b^{ax} + C;\ b > 0, b \neq 1$

11. $\int \ln x\,dx = x\ln x - x + C$

12. $\int \log_b x\,dx = \frac{1}{\ln b}(x\ln x - x) + C$

13. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$

14. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

15. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

16. $\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1 - x^2} + C$

17. $\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1 - x^2} + C$

18. $\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\ln(1 + x^2) + C$

19. $\int \sec^{-1}x\,dx = x\sec^{-1}x - \ln(x + \sqrt{x^2 - 1}) + C$

20. $\int \sinh x\,dx = \cosh x + C$

21. $\int \cosh x\,dx = \sinh x + C$

22. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$

23. $\int \operatorname{csch}^2 x\,dx = -\coth x + C$

24. $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$

25. $\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$

26. $\int \tanh x\,dx = \ln\cosh x + C$

27. $\int \coth x\,dx = \ln|\sinh x| + C$

28. $\int \operatorname{sech} x\,dx = \tan^{-1}\sinh x + C = \sin^{-1}\tanh x + C$

29. $\int \operatorname{csch} x\,dx = \ln|\tanh(x/2)| + C$

Trigonometric Integrals

30. $\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$

31. $\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$

32. $\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$

33. $\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$

34. $\int \tan^2 x\,dx = \tan x - x + C$

35. $\int \cot^2 x\,dx = -\cot x - x + C$

36. $\int \cos^3 x\,dx = -\frac{1}{3}\sin^3 x + \sin x + C$

37. $\int \sin^3 x\,dx = \frac{1}{3}\cos^3 x - \cos x + C$

38. $\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

39. $\int \csc^3 x\,dx = -\frac{1}{2}\csc x \cot x - \frac{1}{2}\ln|\csc x + \cot x| + C$

40. $\int \tan^3 x\,dx = \frac{1}{2}\tan^2 x - \ln|\sec x| + C$

41. $\int \cot^3 x\,dx = -\frac{1}{2}\cot^2 x - \ln|\sin x| + C$

42. $\int \sec^n ax \tan ax\,dx = \frac{1}{na}\sec^n ax + C;\ n \neq 0$

43. $\int \csc^n ax \cot ax\,dx = -\frac{1}{na}\csc^n ax + C;\ n \neq 0$

44. $\int \frac{dx}{1+\sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$

45. $\int \frac{dx}{1-\sin ax} = \frac{1}{a}\tan\left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$

46. $\int \frac{dx}{1+\cos ax} = \frac{1}{a}\tan\frac{ax}{2} + C$

47. $\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\frac{ax}{2} + C$

48. $\int \sin mx \cos nx\,dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C;\ m^2 \neq n^2$

49. $\int \sin mx \sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C;\ m^2 \neq n^2$

50. $\int \cos mx \cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C;\ m^2 \neq n^2$

Reduction Formulas for Trigonometric Functions

51. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x \sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

52. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x \cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

53. $\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx;\ n \neq 1$

54. $\int \cot^n x\,dx = -\frac{\cot^{n-1}x}{n-1} - \int \cot^{n-2}x\,dx;\ n \neq 1$

55. $\int \sec^n x\,dx = \frac{\sec^{n-2}x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx;\ n \neq 1$

56. $\int \csc^n x\,dx = -\frac{\csc^{n-2}x \cot x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2}x\,dx;\ n \neq 1$

57. $\int \sin^m x \cos^n x\,dx = -\frac{\sin^{m-1}x \cos^{n+1}x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2}x \cos^n x\,dx;\ m \neq -n$

58. $\int \sin^m x \cos^n x\,dx = \frac{\sin^{m+1}x \cos^{n-1}x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2}x\,dx;\ m \neq -n$

59. $\int x^n \sin ax\,dx = -\frac{x^n \cos ax}{a} + \frac{n}{a}\int x^{n-1}\cos ax\,dx;\ a \neq 0$

60. $\int x^n \cos ax\,dx = \frac{x^n \sin ax}{a} - \frac{n}{a}\int x^{n-1}\sin ax\,dx;\ a \neq 0$

Integrals Involving $a^2 - x^2$; $a > 0$

61. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

62. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

63. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2 x} + C$

64. $\int x^2\sqrt{a^2-x^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a} + C$

65. $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{1}{x}\sqrt{a^2-x^2} - \sin^{-1}\frac{x}{a} + C$

66. $\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

67. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$

Integrals Involving $x^2 - a^2$; $a > 0$

68. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

69. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$

70. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2 x} + C$

71. $\int x^2\sqrt{x^2-a^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

72. $\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = \ln\left|x+\sqrt{x^2-a^2}\right| - \frac{\sqrt{x^2-a^2}}{x} + C$

73. $\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx = \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + \frac{x}{2}\sqrt{x^2-a^2} + C$

74. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

75. $\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2}\ln\left|\frac{x^2-a^2}{x^2}\right| + C$

Integrals Involving $a^2 + x^2$; $a > 0$

76. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

77. $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x+\sqrt{a^2+x^2}\right) + C$

78. $\int \frac{dx}{x\sqrt{a^2+x^2}} = \frac{1}{a}\ln\left|\frac{a-\sqrt{a^2+x^2}}{x}\right| + C$

79. $\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C$

80. $\int x^2\sqrt{a^2+x^2}\,dx = \frac{x}{8}(a^2+2x^2)\sqrt{a^2+x^2} - \frac{a^4}{8}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

81. $\int \frac{\sqrt{a^2+x^2}}{x^2}\,dx = \ln\left|x+\sqrt{a^2+x^2}\right| - \frac{\sqrt{a^2+x^2}}{x} + C$

82. $\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx = -\frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + \frac{x\sqrt{a^2+x^2}}{2} + C$

83. $\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$

84. $\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} + C$

85. $\int \frac{dx}{x(a^2+x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right) + C$

Integrals Involving $ax \pm b$; $a \neq 0, b > 0$

86. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C;\ n \neq -1$

87. $\int (\sqrt{ax}+b)^n\,dx = \frac{2}{a}\frac{(\sqrt{ax}+b)^{n+2}}{n+2} + C;\ n \neq -2$

88. $\int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}}\tan^{-1}\sqrt{\frac{ax-b}{b}} + C;\ b > 0$

89. $\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C;\ b > 0$

90. $\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$

91. $\int \frac{x^2}{ax+b}\,dx = \frac{1}{2a^3}\left((ax+b)^2 - 4b(ax+b) + 2b^2\ln|ax+b|\right) + C$

92. $\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| + C$

93. $\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(3ax-2b)(ax+b)^{3/2} + C$

94. $\int \frac{x}{\sqrt{ax+b}}\,dx = \frac{2}{3a^2}(ax-2b)\sqrt{ax+b} + C$

95. $\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a^2}\left(\frac{ax+b}{n+2} - \frac{b}{n+1}\right) + C;\ n \neq -1, -2$

96. $\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left|\frac{x}{ax+b}\right| + C$

Integrals with Exponential and Trigonometric Functions

97. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + C$

98. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2} + C$

Integrals with Exponential and Logarithmic Functions

99. $\int \frac{dx}{x\ln x} = \ln|\ln x| + C$

100. $\int x^n\ln x\,dx = \frac{x^{n+1}}{n+1}\left(\ln x - \frac{1}{n+1}\right) + C;\ n \neq -1$

101. $\int xe^x\,dx = xe^x - e^x + C$

102. $\int x^ne^{ax}\,dx = \frac{1}{a}x^ne^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx;\ a \neq 0$

103. $\int \ln^n x\,dx = x\ln^n x - n\int \ln^{n-1}x\,dx$

Miscellaneous Formulas

104. $\int x^n\cos^{-1}x\,dx = \frac{1}{n+1}\left(x^{n+1}\cos^{-1}x + \int \frac{x^{n+1}dx}{\sqrt{1-x^2}}\right);\ n \neq -1$

105. $\int x^n\sin^{-1}x\,dx = \frac{1}{n+1}\left(x^{n+1}\sin^{-1}x - \int \frac{x^{n+1}dx}{\sqrt{1-x^2}}\right);\ n \neq -1$

106. $\int x^n\tan^{-1}x\,dx = \frac{1}{n+1}\left(x^{n+1}\tan^{-1}x - \int \frac{x^{n+1}dx}{x^2+1}\right);\ n \neq -1$

107. $\int \sqrt{2ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C;\ a > 0$

108. $\int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1}\left(\frac{x-a}{a}\right) + C;\ a > 0$

Useful trigonometric identities:

Product-to-sum
$2\cos\theta\cos\varphi = \cos(\theta-\varphi) + \cos(\theta+\varphi)$
$2\sin\theta\sin\varphi = \cos(\theta-\varphi) - \cos(\theta+\varphi)$
$2\sin\theta\cos\varphi = \sin(\theta+\varphi) + \sin(\theta-\varphi)$
$2\cos\theta\sin\varphi = \sin(\theta+\varphi) - \sin(\theta-\varphi)$
$\tan\theta\tan\varphi = \frac{\cos(\theta-\varphi)-\cos(\theta+\varphi)}{\cos(\theta-\varphi)+\cos(\theta+\varphi)}$
$\prod_{k=1}^{n}\cos\theta_k = \frac{1}{2^n}\sum_{e\in S}\cos(e_1\theta_1+\cdots+e_n\theta_n)$ where $S=\{1,-1\}^n$

Sum-to-product
$\sin\theta\pm\sin\varphi = 2\sin\left(\frac{\theta\pm\varphi}{2}\right)\cos\left(\frac{\theta\mp\varphi}{2}\right)$
$\cos\theta+\cos\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$
$\cos\theta-\cos\varphi = -2\sin\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$

Sine	$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta\pm\cos\alpha\sin\beta$
Cosine	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta\mp\sin\alpha\sin\beta$
Tangent	$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$
Cosecant	$\csc(\alpha\pm\beta) = \frac{\sec\alpha\sec\beta\csc\alpha\csc\beta}{\sec\alpha\csc\beta\pm\csc\alpha\sec\beta}$
Secant	$\sec(\alpha\pm\beta) = \frac{\sec\alpha\sec\beta\csc\alpha\csc\beta}{\csc\alpha\csc\beta\mp\sec\alpha\sec\beta}$
Cotangent	$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta\mp 1}{\cot\beta\pm\cot\alpha}$
Arcsine	$\arcsin x\pm\arcsin y = \arcsin\left(x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right)$
Arccosine	$\arccos x\pm\arccos y = \arccos\left(xy\mp\sqrt{(1-x^2)(1-y^2)}\right)$
Arctangent	$\arctan x\pm\arctan y = \arctan\left(\frac{x\pm y}{1\mp xy}\right)$
Arccotangent	$\operatorname{arccot} x\pm\operatorname{arccot} y = \operatorname{arccot}\left(\frac{xy\mp 1}{y\pm x}\right)$