The duration of this exam is 3 hours. There are 5 questions and the students must work 4 of them. If more than 4 questions are attempted, the student must specify which 4 are be graded. If not specified, the first 4 questions will be graded. Each question is 25 points and you need 70 points to pass this exam. Good Luck!

Problem 1.

(a) Using Newton Raphson method, find the first two iteration values of the root for

$$y = 3x^4 - 7x^2 + 6x + 1 \tag{1}$$

Assume the initial guess is $x = 0$.

(b) Determine $\|A\|_1$, $\|A\|_2$, and $\|A\|_\infty$ for

$$[A] = \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix} \tag{2}$$

Problem 2.

(a) Solve the following system of equations using LU factorization

$$2x_1 - 6x_2 - x_3 = -38 \quad (3)$$

$$-3x_1 - x_2 + 7x_3 = -34 \quad (4)$$

$$-8x_1 + x_2 - 2x_3 = -40 \quad (5)$$

(b) Use the Gauss-Seidel method to solve the following system until the percent relative error falls below $\epsilon_s = 5\%$:

$$\begin{bmatrix} 0.8 & -0.4 & 0 \\ -0.4 & 0.8 & -0.4 \\ 0 & -0.4 & 0.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} \quad (6)$$

Problem 3. You perform experiments and determine the following values of heat capacity c at various temperatures T for a gas:

T	-50	-30	0	60	90	110
c	1250	1280	1350	1480	1580	1700

Use regression to determine a model to predict c as a function of T.

problem 4.

(a) Numerically evaluate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial^2 f}{\partial x \partial y}$ for

$$f(x, y) = 3xy + 3x - x^3 - 3y^3 \tag{7}$$

at $x = y = 1$ with $\Delta x = \Delta y = 0.0001$ (any numerical approach is accepted.)

(b) The total mass of a variable density rod is given by

$$m = \int_0^L \rho(x) A_c(x) dx \tag{8}$$

where m is mass, $\rho(x)$ is density, $A_c(x)$ is cross-sectional area, x is distance along the rod, and L is the total length of the rod. The following data have been measured for a 20-m length rod. Determine the mass in grams to the best possible accuracy.

x	m	0	4	6	8	12	16	20
ρ	g/cm^3	4.00	3.95	3.89	3.80	3.60	3.41	3.30
A_c	cm^2	100	103	106	110	120	133	150

Problem 5. Given

$$\frac{dx_1}{dt} = 999x_1 + 1999x_2 \tag{9}$$

$$\frac{dx_2}{dt} = -1000x_1 - 2000x_2 \tag{10}$$

If $x_1(0) = x_2(0) = 1$, obtain a solution from t = 0 to 0.15 using a step size of 0.05 with the (a) explicit and (b) implicit Euler methods.