# Syllabus for Numerical Methods Topics of the Ph.D. Qualifying Exam
## August 2023 Version

### Department of Mechanical Engineering & Engineering Science

The student is expected to be knowledgeable in the topics listed below. The references given at the end of this syllabus may be useful for the student in preparing for the exam.

**Topics:**

1. Numerical Differentiation and Integration
   - Finite differences
   - Newton-Cotes formulae for integration
   - Gauss-Legendre quadrature

2. Interpolation
   - Divided differences
   - Lagrange interpolation

3. Nonlinear Systems of Equations
   - Newton-Raphson method
   - Bisection method
   - Modified Newton's method

4. Linear Algebra
   - Elementary Matrix Operations
   - Matrix Norms
   - Rank of a matrix, determinant of a matrix, Eigenvalues and Eigenvectors
   - Positive definite matrices, orthogonal matrices, similarity transformations
   - Linear regression
   - Direct methods for solving systems of equations: Gaussian elimination, LU decomposition, Cholesky factorization
   - Iterative methods for solving systems of equations: Jacobi's method, Gauss-Seidel and SOR
   - Eigenvalues and Eigenvectors: Power method and Jacobi's method

5. Numerical Solutions to Systems of First-Order ODEs:
   - Single-step methods: Taylor Series Method, Euler methods (Forward, Backward and Modified), and Runge-Kutta Methods
   - Multistep methods: Adams-Bashforth method, Adams-Moulton method
   - Adaptive time-stepping techniques for single-step methods (Interval halving and Runge-Kutta-Fehlberg methods)
   - General understanding of stability and stiffness of ODEs
   - Two-point boundary value problems (shooting method)

**References:**

1. C. Pozrikidis. Numerical Computation in Science and Engineering, 2nd edition, Oxford University Press, 2008.
2. Joe D. Hoffman. Numerical Methods for Engineers and Scientists, 2nd edition, Taylor & Francis, 2001.