Dynamics Qualifying Exam Sample

Instructions: Complete the following five problems worth 20 points each. No material other than a calculator and pen/pencil can be used in the exam. A passing grade is approximately 70 points. If you do not understand something, <u>make reasonable assumptions and state them clearly</u>. This will be considered in the grading.



KINEMATICS				
Particle Rectilinear Moti	ion			
Variable a	Constant $a = a_c$			
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$			
$v = \frac{ds}{dt}$	$s = s_{\odot} + v_0 t + \frac{1}{2}a_c t^2$			
a ds = v dv	$v^2 = v_0^2 + 2a_c(s - s_0)$			
Particle Curvilinear Mot	ion			
x, y, z Coordinates	r, θ , z Coordinates			
$v_x = \dot{x}$ $a_x = \ddot{x}$	$v_r = \dot{r}$ $a_r = \ddot{r} - r\dot{\theta}^2$			
$v_y = \dot{y}$ $a_y = \ddot{y}$	$v_{\theta} = r\dot{\theta} a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta}$			
$v_z = \dot{z}$ $a_z = \ddot{z}$	$v_z = \dot{z}$ $a_z = \ddot{z}$			
n, t, b Coordinates				
$v = \dot{s}$ $a_{\cdot} = \dot{v} = v^{-1}$	dv			
	ds			
$a_n = \frac{v^2}{\rho}$	$=\frac{\left[1+(dy/dx)^{2}\right]^{3/2}}{\left[d^{2}y/dx^{2}\right]}$			
P Relative Motion	$ a^{-}y/ax^{-} $			
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \mathbf{a}_B = 1$	$\mathbf{a}_A + \mathbf{a}_{B/A}$			
Rigid Body Motion Abo	ut a Fixed Axis			
Variable α	Constant $\alpha = \alpha_c$			
$\alpha = \frac{d\omega}{d\omega}$	$\omega = \omega \pm \alpha t$			
$a = \frac{dt}{dt}$	$\omega - \omega_0 + \alpha_c \iota$			
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$			
$\omega d\omega = \alpha d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$			
For Point P				
$s = \theta r$ $v = \omega r$ $a_t =$	$\alpha r a_n = \omega^2 r$			
Relative General Plane	Motion—Translating Axes			
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \mathbf{a}_B$	$= \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$			
Relative General Plane Motion-Trans. and Rot. Axis				
$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} \times (\mathbf{v}_{B/A})_{xyz}$				
$\mathbf{a}_B = \mathbf{a}_A + \Omega \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) +$				
$2\Omega imes (\mathbf{v}_{B/A})_{xyz} imes (\mathbf{a}_{B/A})_{xyz}$				
KINETICS				
Second Moment of Mass $I = \int r^2 dm$				
Parallel-Axis Theorem	$I = I_G + md^2$			
Radius of Gyration	$k = \sqrt{\frac{I}{m}}$			

Particle $\Sigma \mathbf{F} = m\mathbf{a}$ Rigid Body $\Sigma F_x = m(a_G)_x$ $(Plane Motion)$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G \alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$ Principle of Work and Energy $T_1 + U_{1-2} = T_2$ Kinetic Energy $Particle$ $T = \frac{1}{2}mv^2$ Rigid Body $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Work $Variable$ force $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_w = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F}_{\mathcal{P}} \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = \pm \frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $mv_1 \pm \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 \pm \Sigma \int \mathbf{F} dt = m(v_G)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Equations of Motion				
$Tarriele$ $\Sigma \mathbf{F} = m\mathbf{a}$ Rigid Body $\Sigma F_x = m(a_G)_x$ $(Plane Motion)$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = l_G \alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$ Principle of Work and Energy $T_1 + U_{1-2} = T_2$ Kinetic Energy $Particle$ $T = \frac{1}{2}mv^2$ Rigid Body $T = \frac{1}{2}mv^2$ (Plane Motion) $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Work $Variable force$ $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_w = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \varphi \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = \pm \frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $mv_1 \pm \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 \pm \Sigma \int \mathbf{F} dt = m(v_G)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$ $\Sigma(syst. \mathbf{H})_2$	Dartiele				
Augus Dody $\Sigma F_x = m(a_G)_x$ $\Sigma M_G = I_G \alpha \text{ or } \Sigma M_P = \Sigma(\mathcal{M}_k)_P$ Principle of Work and Energy $T_1 + U_{1-2} = T_2$ $\Sigma M_G = I_G \alpha \text{ or } \Sigma M_P = \Sigma(\mathcal{M}_k)_P$ Rinetic Energy Particle $T = \frac{1}{2}mv^2$ Rigid Body (Plane Motion) $T = \frac{1}{2}mv^2$ Work Variable force $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_w = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = \pm \frac{1}{2}ks^2$ Principle of Linear Impulse and Momentum ParticleMatricle $mv_1 \pm \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 \pm \Sigma \int \mathbf{F} dt = m(v_G)_2$ Conservation of Linear Momentum $\Sigma(syst. mv)_1 = \Sigma(syst. mv)_2$ Coefficient of Restitution where $H_O = (d)(mv)$ Particle $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ Where $H_O = (d)(mv)$ Rigid Body $(\mathbf{H}_G)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ Where $H_O = I_G \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Rigid Rody	$\sum F = m(a_{-})$			
Principle of Work and Energy $\Sigma M_G = I_G \alpha$ or $\Sigma M_P = \Sigma(M_k)_P$ Principle of Work and Energy $T_1 + U_{1-2} = T_2$ Kinetic Energy Particle $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Rigid Body (Plane Motion) $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Work Variable force $U_F = \int F \cos \theta ds$ Constant force Weight $U_F = (F_c \cos \theta) \Delta s$ Weight $U_W = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$ Principle of Linear Impulse and Momentum ParticleParticle $mv_1 + \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 + \Sigma \int \mathbf{F} dt = m(v_G)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum $2(syst. mv)_1 = \Sigma(syst. mv)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum $Particle$ Particle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	(Plane Motion)	$\sum F = m(a_G)_x$			
Principle of Work and Energy $T_1 + U_{1-2} = T_2$ Kinetic Energy Particle $T = \frac{1}{2}mv^2$ Rigid Body (Plane Motion) $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Work Variable force $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_w = -W \Delta y$ Spring $U_s = -(\frac{1}{2} ks_2^2 - \frac{1}{2} ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2} ks^2$ Principle of Linear Impulse and Momentum $Particle$ $mv_1 + \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 + \Sigma \int \mathbf{F} dt = m(v_G)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum $Particle$ Particle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ Where $H_O = I_G \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	(1 mile monor)	$\sum M_{G} = I_{G} \alpha \text{ or } \Sigma M_{P} = \Sigma(\mathcal{M}_{L})_{P}$			
$T_{1} + U_{1-2} = T_{2}$ Kinetic Energy Particle $T = \frac{1}{2}mv^{2}$ Rigid Body (Plane Motion) $T = \frac{1}{2}mv_{G}^{2} + \frac{1}{2}I_{G}\omega^{2}$ Work Variable force $U_{F} = \int F \cos \theta ds$ Constant force $U_{F} = (F_{c} \cos \theta) \Delta s$ Weight $U_{W} = -W \Delta y$ Spring $U_{s} = -(\frac{1}{2}ks_{2}^{2} - \frac{1}{2}ks_{1}^{2})$ Couple moment $U_{M} = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_{1} + V_{1} = T_{2} + V_{2}$ Potential Energy $V = V_{g} + V_{e}, \text{ where } V_{g} = \pm Wy, V_{e} = +\frac{1}{2}ks^{2}$ Principle of Linear Impulse and Momentum Particle $mv_{1} + \sum \int \mathbf{F} dt = mv_{2}$ Rigid Body $m(v_{G})_{1} + \sum \int \mathbf{F} dt = m(v_{G})_{2}$ Coefficient of Restitution $e = \frac{(v_{B})_{2} - (v_{A})_{2}}{(v_{A})_{1} - (v_{B})_{1}}$ Principle of Angular Impulse and Momentum Particle $(\mathbf{H}_{O})_{1} + \sum \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = (d)(mv)$ Rigid Body $(\mathbf{H}_{O})_{1} + \sum \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = I_{G}\omega$ $(\mathbf{H}_{O})_{1} + \sum \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = I_{O}\omega$ Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_{1} = \Sigma(\text{syst. } \mathbf{H})_{2}$	Principle of Work	and Energy			
Kinetic Energy Particle $T = \frac{1}{2}mv^2$ Rigid Body (Plane Motion) $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Work Variable force $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_W = -W \Delta y$ Spring $U_s = -(\frac{1}{2} ks_2^2 - \frac{1}{2} ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F}\wp \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = +\frac{1}{2} ks^2$ Principle of Linear Impulse and Momentum ParticleM(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2Conservation of Energy time $V_g = \pm (w_g)_2 - (w_g)_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = (w_g)_2 - (w_g)_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = (w_g)_2$ Principle of Linear Impulse and Momentum $2(syst. mv)_1 = \Sigma(syst. mv)_2$ Coefficient of Restitution $Particle$ Particle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body $(Plane motion)$ $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	$T_1 + U_{1-2} = T_2$				
Particle $T = \frac{1}{2}mv^2$ Rigid Body (Plane Motion) $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Work Variable force $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_w = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks^2 - \frac{1}{2}ks^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = +\frac{1}{2}ks^2$ Principle of Linear Impulse and Momentum ParticleParticle $m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. mv)_1 = \Sigma(syst. mv)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum $Particle$ Migid Body $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Kinetic Energy				
Rigid Body (Plane Motion) $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ Work Variable force $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_W = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v}$ $\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = \pm \frac{1}{2}ks^2$ Principle of Linear Impulse and Momentum Particle $mv_1 + \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 + \Sigma \int \mathbf{F} dt = m(v_G)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum $\Sigma(syst. mv)_1 = \Sigma(syst. mv)_2$ Rigid Body $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_G = I_G \omega$ Rigid Body $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Particle	$T = \frac{1}{2}mv^2$			
$\begin{array}{l c c c c c c c c c c c c c c c c c c c$	Rigid Body	$T = 1 \dots 2 + 1 T = 2$			
Work Variable force $U_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_W = -W \Delta y$ Spring $U_s = -(\frac{1}{2} ks_2^2 - \frac{1}{2} ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v}$ $\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2} ks^2$ Principle of Linear Impulse and Momentum Particle $M(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum $Particle$ $M_G dt = (\mathbf{H}_G)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	(Plane Motion)	$I = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$			
Variable force $D_F = \int F \cos \theta ds$ Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_w = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v} \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = \pm \frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $mv_1 \pm \sum \int \mathbf{F} dt = mv_2$ Rigid Body $\mathcal{K}(syst. mv)_1 = \Sigma(syst. mv)_2$ Coefficient of Restitution $Particle$ $(\mathbf{H}_O)_1 \pm \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body(H_O)_1 \pm \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 \pm \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Work	$U = \int \Gamma_{aaa} \partial_{a} da$			
Constant force $U_F = (F_c \cos \theta) \Delta s$ Weight $U_W = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v}$ $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v}$ $\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $mv_1 + \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 + \Sigma \int \mathbf{F} dt = m(v_G)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	variable force	$U_F = \int F \cos \theta ds$			
Weight $U_W = -W \Delta y$ Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v}$ $\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = \pm \frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $m\mathbf{v}_1 \pm \sum \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 \pm \sum \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. mv)_1 = \Sigma(syst. mv)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 \pm \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body $(\mathbf{H}_G)_1 \pm \sum \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ Where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 \pm \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Constant force	$U_F = (F_c \cos \theta) \Delta s$			
Spring $U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v}$ $\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy, V_e = \pm \frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $mv_1 \pm \Sigma \int \mathbf{F} dt = mv_2$ Rigid Body $\mathcal{K}(\mathbf{y}_G)_1 \pm \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Coefficient of Restitution $\mathcal{P} = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Weight	$U_W = -W \Delta y$			
Couple moment $U_M = M \Delta \theta$ Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v}$ $\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = \pm \frac{1}{2} ks^2$ Principle of Linear Impulse and MomentumParticle $m\mathbf{v}_1 \pm \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 \pm \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. mv)_1 = \Sigma(syst. mv)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body($\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 \pm \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Spring	$U_{s} = -(\frac{1}{2}ks_{2}^{2} - \frac{1}{2}ks_{1}^{2})$			
Power and Efficiency $P = \frac{dU}{dt} = \mathbf{F} \wp \mathbf{v} \boldsymbol{\epsilon} = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$ Conservation of Energy Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e, \text{ where } V_g = \pm Wy, V_e = \pm \frac{1}{2} ks^2$ Principle of Linear Impulse and Momentum Particle $m\mathbf{v}_1 + \sum \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 + \sum \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$ Coefficient of Restitution $Particle \qquad (\mathbf{H}_O)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$	Couple moment	$U_M = M \Delta \theta$			
$P = \frac{1}{dt} = \mathbf{F}_{\mathcal{O}}\mathbf{v} \boldsymbol{\epsilon} = \frac{1}{P_{in}} = \frac{1}{U_{in}}$ Conservation of Energy Theorem $T_{1} + V_{1} = T_{2} + V_{2}$ Potential Energy $V = V_{g} + V_{e}, \text{ where } V_{g} = \pm Wy, V_{e} = +\frac{1}{2}ks^{2}$ Principle of Linear Impulse and Momentum Particle $m\mathbf{v}_{1} + \Sigma \int \mathbf{F} dt = m\mathbf{v}_{2}$ Rigid Body $m(\mathbf{v}_{G})_{1} + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_{G})_{2}$ Conservation of Linear Momentum $\Sigma(\text{syst. } m\mathbf{v})_{1} = \Sigma(\text{syst. } m\mathbf{v})_{2}$ Coefficient of Restitution $Particle$ $(\mathbf{H}_{O})_{1} + \Sigma \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_{O})_{1} + \Sigma \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = I_{O}\omega$ Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_{1} = \Sigma(\text{syst. } \mathbf{H})_{2}$	Power and Efficie dU –	$P_{\text{out}} U_{\text{out}}$			
Image matrix matrix matrix Theorem $T_1 + V_1 = T_2 + V_2$ Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. m\mathbf{v})_1 = \Sigma(syst. m\mathbf{v})_2$ $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum Particle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	$P = \frac{1}{dt} = \mathbf{F} \wp \mathbf{v}$	$\epsilon = \frac{-out}{P_{in}} = \frac{-out}{U_{in}}$			
$T_{1} + V_{1} = T_{2} + V_{2}$ Potential Energy $V = V_{g} + V_{e}, \text{ where } V_{g} = \pm Wy, V_{e} = +\frac{1}{2}ks^{2}$ Principle of Linear Impulse and Momentum Particle $mv_{1} + \sum \int \mathbf{F} dt = mv_{2}$ Rigid Body $m(v_{G})_{1} + \sum \int \mathbf{F} dt = m(v_{G})_{2}$ Conservation of Linear Momentum $\Sigma(\text{syst. } mv)_{1} = \Sigma(\text{syst. } mv)_{2}$ Coefficient of Restitution $e = \frac{(v_{B})_{2} - (v_{A})_{2}}{(v_{A})_{1} - (v_{B})_{1}}$ Principle of Angular Impulse and Momentum Particle $(\mathbf{H}_{O})_{1} + \sum \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_{O})_{1} + \sum \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{G} = I_{G}\omega$ $(\mathbf{H}_{O})_{1} + \sum \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = I_{O}\omega$ Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_{1} = \Sigma(\text{syst. } \mathbf{H})_{2}$	Conservation of E	Energy Theorem			
Potential Energy $V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$ Principle of Linear Impulse and MomentumParticle $m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. m\mathbf{v})_1 = \Sigma(syst. m\mathbf{v})_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	$T_1 + V_1 = T_2 + V_1$	2			
$V = V_g + V_e, \text{ where } V_g = \pm Wy, V_e = \pm ks^2$ Principle of Linear Impulse and Momentum Particle $mv_1 + \sum \int \mathbf{F} dt = mv_2$ Rigid Body $m(v_G)_1 + \sum \int \mathbf{F} dt = m(v_G)_2$ Conservation of Linear Momentum $\Sigma(\text{syst. } mv)_1 = \Sigma(\text{syst. } mv)_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum Particle $(\mathbf{H}_O)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$	Potential Energy	1.2			
Principle of Linear Impulse and MomentumParticle $m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. m\mathbf{v})_1 = \Sigma(syst. m\mathbf{v})_2$ Coefficient of Restitution $\mathcal{P} = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	$V = V_g + V_e$, whe	re $V_g = \pm Wy, V_e = \pm \frac{1}{2}ks^2$			
Particle $m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$ Rigid Body $m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. m\mathbf{v})_1 = \Sigma(syst. m\mathbf{v})_2$ $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum Particle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Principle of Linea	r Impulse and Momentum			
Rigid Body $m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$ Conservation of Linear Momentum $\Sigma(syst. m\mathbf{v})_1 = \Sigma(syst. m\mathbf{v})_2$ $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and Momentum Particle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Particle	$m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$			
Conservation of Linear Momentum $\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$ Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$	Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$			
$\Sigma(\text{syst. } m\mathbf{v})_{1} = \Sigma(\text{syst. } m\mathbf{v})_{2}$ Coefficient of Restitution $e = \frac{(v_{B})_{2} - (v_{A})_{2}}{(v_{A})_{1} - (v_{B})_{1}}$ Principle of Angular Impulse and Momentum Particle $(\mathbf{H}_{O})_{1} + \Sigma \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_{G})_{1} + \Sigma \int \mathbf{M}_{G} dt = (\mathbf{H}_{G})_{2}$ where $H_{G} = I_{G}\omega$ $(\mathbf{H}_{O})_{1} + \Sigma \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = I_{O}\omega$ Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_{1} = \Sigma(\text{syst. } \mathbf{H})_{2}$	Conservation of L	inear Momentum			
Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ Principle of Angular Impulse and MomentumParticle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ Where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	$\Sigma(\text{syst. }m\mathbf{v})_1 = \Sigma(s)$	$(w_{\tau})_{2} = (w_{\tau})_{2}$			
(\mathcal{H}_A)1 (\mathcal{H}_B)1ParticleParticle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_B)_2 - (v_A)_2}$				
Particle $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Principle of Angular Impulse and Momentum				
where $H_O = (d)(mv)$ Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \sum \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \sum \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$	Particle	$ (\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2 $			
Rigid Body (Plane motion) $(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$ where $H_G = I_G \omega$ $(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum $\Sigma(syst. \mathbf{H})_1 = \Sigma(syst. \mathbf{H})_2$		where $H_O = (d)(mv)$			
(<i>Plane motion</i>) (<i>Plane motion</i>) where $H_G = I_G \omega$ (\mathbf{H}_O) ₁ + $\Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$ where $H_O = I_O \omega$ Conservation of Angular Momentum Σ (syst. \mathbf{H}) ₁ = Σ (syst. \mathbf{H}) ₂	Divid Dodu	$(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$			
$(\mathbf{H}_{O})_{1} + \sum \int \mathbf{M}_{O} dt = (\mathbf{H}_{O})_{2}$ where $H_{O} = I_{O}\omega$ Conservation of Angular Momentum $\Sigma(\text{syst. }\mathbf{H})_{1} = \Sigma(\text{syst. }\mathbf{H})_{2}$	(Plane motion)	where $H_G = I_G \omega$			
$\sum (syst. \mathbf{H})_1 = \sum (syst. \mathbf{H})_2$	(1 tune motion)	$(\mathbf{H}_{0})_{1} + \Sigma \int \mathbf{M}_{0} dt = (\mathbf{H}_{0})_{2}$			
Conservation of Angular Momentum $\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$		where $H_Q = I_Q \omega$			
$\Sigma(\text{syst. }\mathbf{H})_1 = \Sigma(\text{syst. }\mathbf{H})_2$	Conservation of Angular Momentum				

Fundamental Equations of Dynamics

Laplace Transform Tables

TABLE 2-1 La	place Trans	sform Pairs
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	f(t)	F(s)
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t and the second second	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \qquad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	t^n (<i>n</i> = 1, 2, 3,)	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!}t^{n-1}e^{-at} \qquad (n=1,2,3,\ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at}$ (<i>n</i> = 1, 2, 3,)	$\frac{n!}{(s+a)^{n+1}}$
10	sin <i>wt</i>	$\frac{\omega}{s^2+\omega^2}$
11	cos wt	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt}-ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab} \left[1 + \frac{1}{a-b} (be^{-at} - ae^{-bt}) \right]$	$\frac{1}{s(s+a)(s+b)}$

	f(t)	F(s)
18	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Problem 1: Consider the piston-crank system shown below. The mass of the piston is 1.5 kg and it is constrained to move linearly in the direction shown. The angular speed of the crank AB is a constant 5.30 rad/sec in the direction shown.



- (a) In the position shown, calculate the angular velocity (magnitude and direction) of link BD and the velocity of the piston.
- (b) Find the force on link BD assuming that the link BD has negligible mass/inertia.

Problem 2: An engineer at the National Institute of Standards and technology (NIST) is testing a new device to measure the static friction coefficient between two materials. Her device operates as follows: (1) A sample of one of the materials to be tested is mounted by strong epoxy to a rotating disk to form a slip-surface as shown; (2) a sample of mass *m* of the second material is placed on the slip-surface at a known distance L from the center of rotation; (3) the rotary table is accelerated from zero angular speed at a rate of α rad/sec²; (4) the angular speed ω_{slip} at which slipping first occurs is recorded.



- (a) Find the mathematical relationship between the static friction coefficient μ_s and the other parameters α , ω_{slip} and *m* and the acceleration of gravity *g*.
- (b) Sketch on the diagram below the direction you would observe the block to initially slip relative to the surface of the disk. (Hint: think very carefully about the components of the acceleration and the conditions under which slipping will occur. Consider limiting cases of very high and very low α and ω .)

Problem 3: A uniform door of mass 20 kg can be treated as a uniform plate having the dimensions shown. If it is connected to a torsional spring at A which has a stiffness of 80 N-m/rad, determine the required initial twist in the spring in radians so that the door has an angular velocity of 12 rad/sec when it closes at $\theta=0^{\circ}$ after being opened to $\theta=90^{\circ}$ and released from rest. (Hint: For a torsional spring M=k θ where k is the stiffness and θ is the angle of twist.)



Problem 4: The motions of a violin string can be modeled as a string stretched between two walls as shown below in Figure 1(a). The string has a constant tension T and a mass per unit length γ . The displacement of the string at any time t is given is y(x,t). If the motions are assumed to be small the tension in the string remains constant (T) and the equations of motion for the string can be derived by considering the differential element shown in Figure 1(b).



Figure: (a) Diagram of a vibrating string stretched between two walls; (b) a differential element of the string of length dx.

- (a) Assuming small motions write down the approximate relationship between the angle of the string $\theta(x,t)$ and the displacement of the string y(x,t).
- (b) Using a first order Taylor series expansion with dx as the small variable, write down an approximate relation for $\theta(x+dx,t)$ in terms of y(x,t).
- (c) Assuming small motions and ignoring gravity, apply Netwon's second law $(\Sigma F=ma)$ to the differential element in Figure (b) to obtain the equation of motion for the string. Hint: (1) ΣF is the sum of the forces on the element; (2) the mass of the differential element dm is the length dx multiplied by the mass per unit length γ ; and (3) the acceleration is $a = \partial^2 y / \partial t^2$.
- (d) Discuss <u>in general</u> how to solve the equation of motion for the vibrating string. What are the boundary conditions? What happens if the guitarist increases the tension in the string – show the effect mathematically and describe what happens in words?

Poblem 5: The 30 kg electric motor shown below is supported by four springs each having a stiffness of 200 N/m. If the rotor is imbalanced so that its effect is equivalent to a 4 kg mass located 60 mm from the axis of rotation. The non-dimensional damping ratio of the system is $\zeta = 0.15 (\zeta = c/(2(km)^{1/2}))$.



- (a) Determine the amplitude of vibration when the rotor is turning at 95.5 revolutions per minute (RPM).
- (b) A new technician arrives on Monday morning and gets a shock as he ramps the motor speed up from rest to a final speed of 150 RPM in ten seconds. What does the technician observe? Give some quantitative estimates of what ocurs and discuss the most major limitation of your analysis.