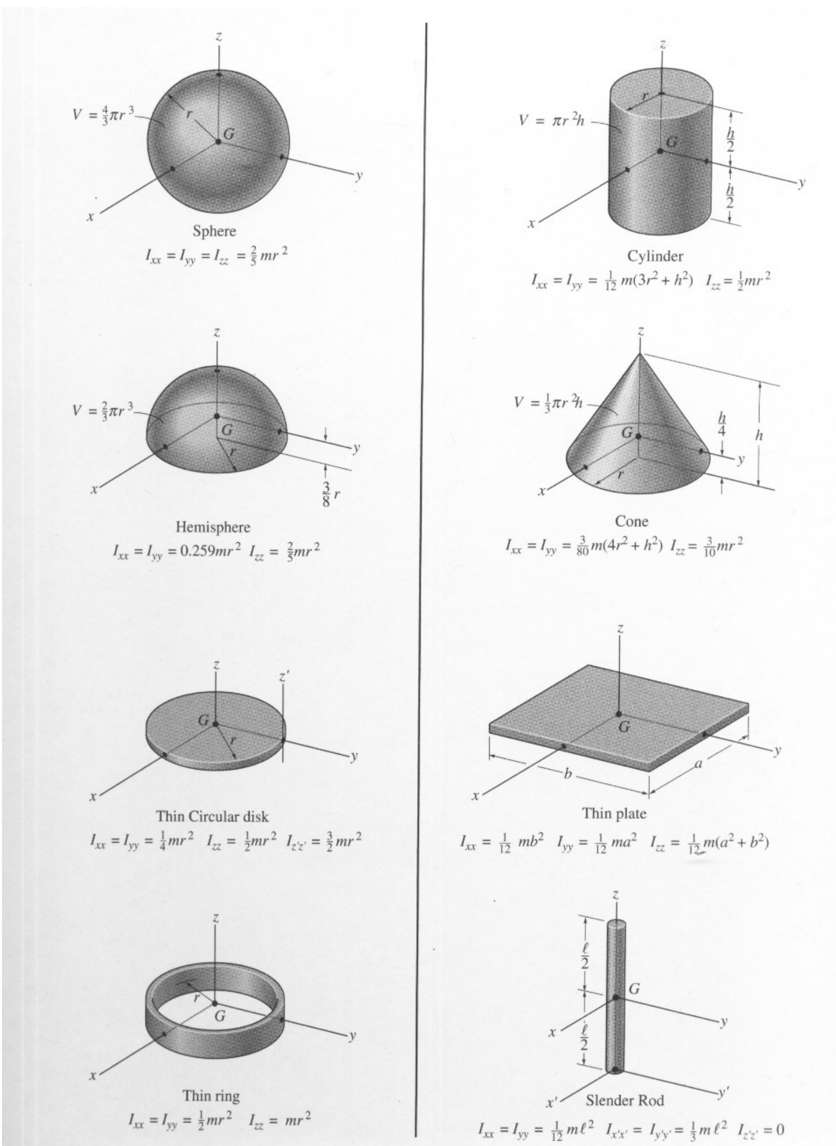
Dynamics Qualifying Exam
Sample

Instructions: Complete the following five problems worth 20 points each. No material other than a calculator and pen/pencil can be used in the exam. A passing grade is approximately 70 points. If you do not understand something, make reasonable assumptions and state them clearly. This will be considered in the grading.

2nd Moment of Mass for Homogeneous Solids
(Also called Mass Moment of Inertia)



Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$ | $a_t = \dot{v} = v\dfrac{dv}{ds}$

$a_n = \dfrac{v^2}{\rho} \quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant $\alpha = \alpha_c$*
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} \times (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} \times (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Second Moment of Mass	$I = \int r^2\,dm$
Parallel-Axis Theorem	$I = I_G + md^2$
Radius of Gyration	$k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \qquad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

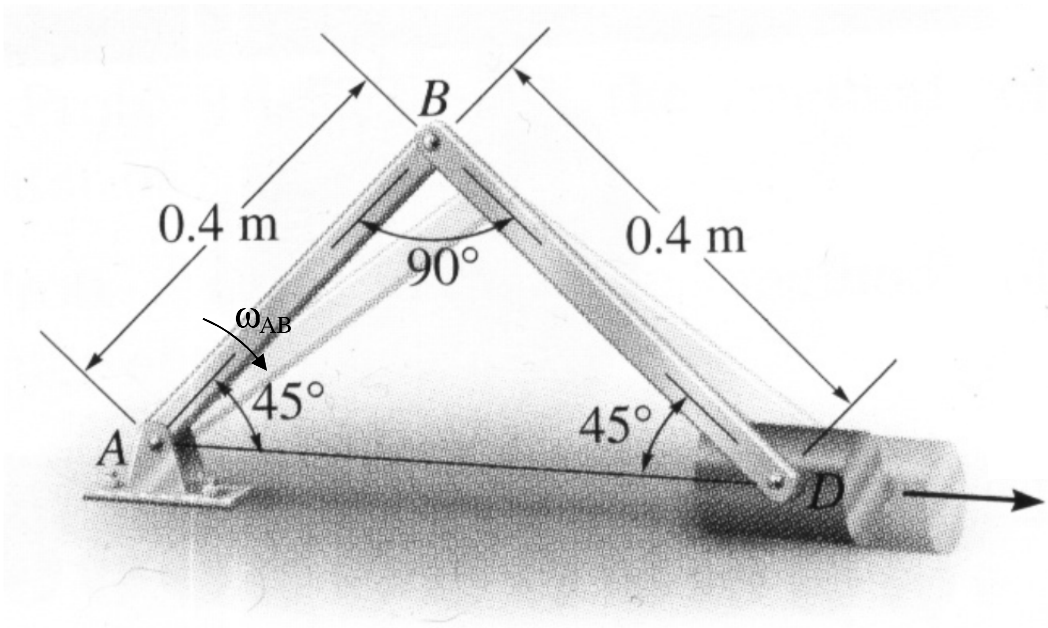
$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

Laplace Transform Tables

TABLE 2–1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!}t^{n-1}e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt}-ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
18	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

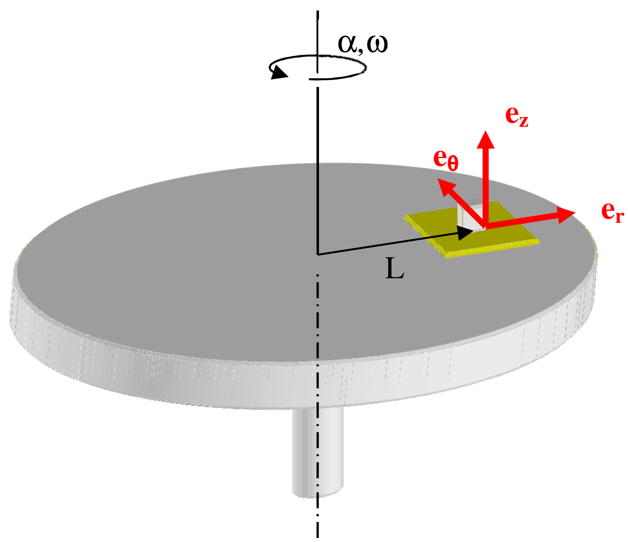
Problem 1: Consider the piston-crank system shown below. The mass of the piston is 1.5 kg and it is constrained to move linearly in the direction shown. The angular speed of the crank AB is a constant 5.30 rad/sec in the direction shown.



(a) In the position shown, calculate the angular velocity (magnitude and direction) of link BD and the velocity of the piston.

(b) Find the force on link BD assuming that the link BD has negligible mass/inertia.

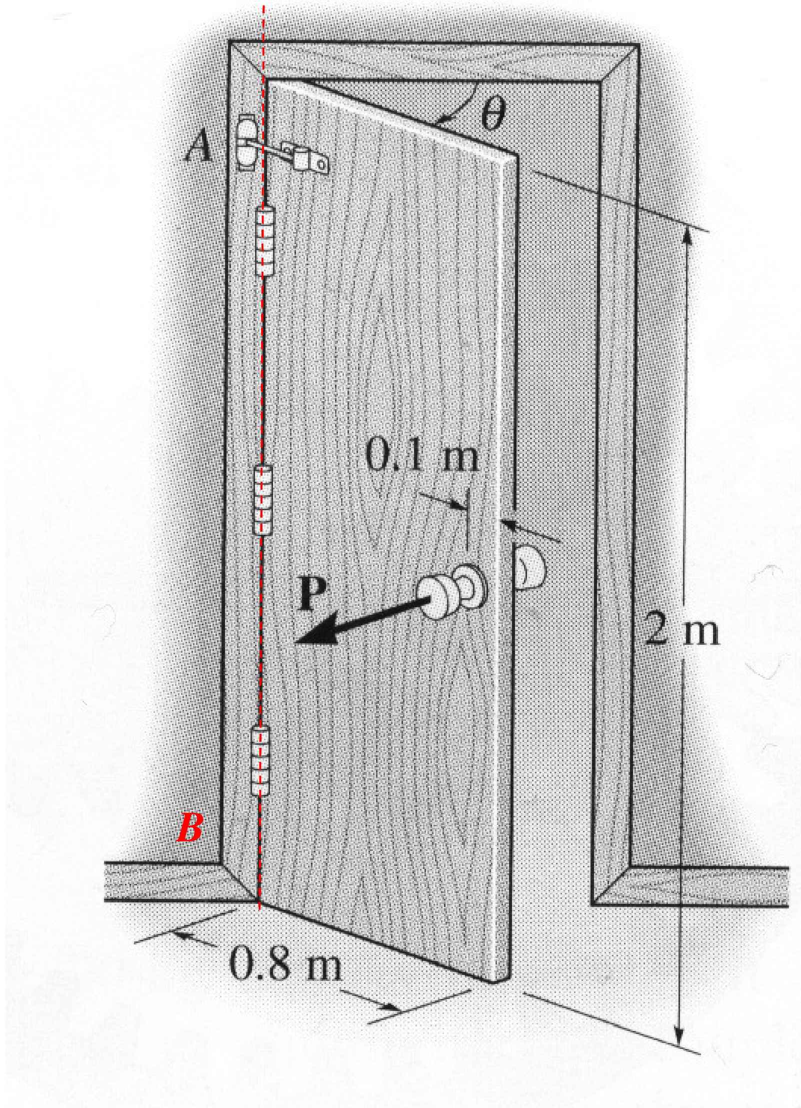
Problem 2: An engineer at the National Institute of Standards and technology (NIST) is testing a new device to measure the static friction coefficient between two materials. Her device operates as follows: (1) A sample of one of the materials to be tested is mounted by strong epoxy to a rotating disk to form a slip-surface as shown; (2) a sample of mass *m* of the second material is placed on the slip-surface at a known distance L from the center of rotation; (3) the rotary table is accelerated from zero angular speed at a rate of α rad/sec^2; (4) the angular speed ω_{slip} at which slipping first occurs is recorded.



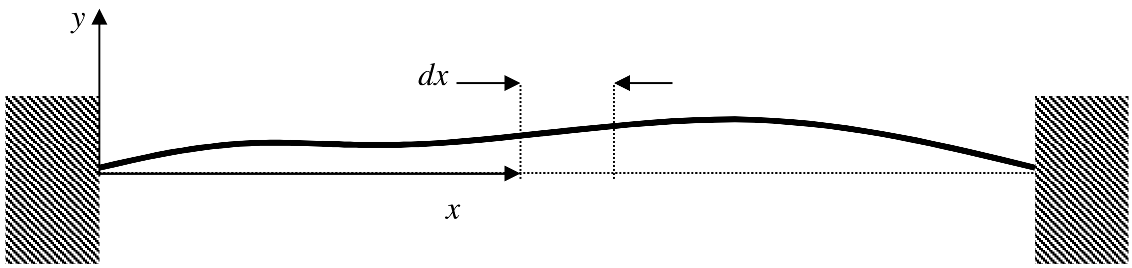
(a) Find the mathematical relationship between the static friction coefficient μ_s and the other parameters α, ω_{slip} and m and the acceleration of gravity g.

(b) Sketch on the diagram below the direction you would observe the block to initially slip relative to the surface of the disk. (Hint: think very carefully about the components of the acceleration and the conditions under which slipping will occur. Consider limiting cases of very high and very low α and ω.)

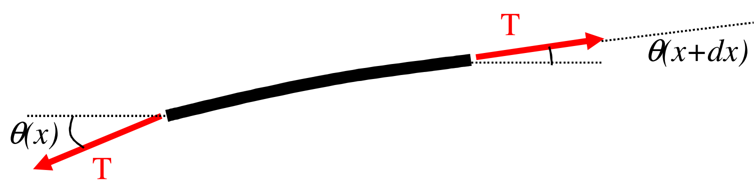
Problem 3: A uniform door of mass 20 kg can be treated as a uniform plate having the dimensions shown. If it is connected to a torsional spring at A which has a stiffness of 80 N-m/rad, determine the required initial twist in the spring in radians so that the door has an angular velocity of 12 rad/sec when it closes at $\theta=0^{\circ}$ after being opened to $\theta=90^{\circ}$ and released from rest. (Hint: For a torsional spring $M=k\theta$ where k is the stiffness and θ is the angle of twist.)



Problem 4: The motions of a violin string can be modeled as a string stretched between two walls as shown below in Figure 1(a). The string has a constant tension *T* and a mass per unit length γ. The displacement of the string at any time *t* is given is *y(x,t)*. If the motions are assumed to be small the tension in the string remains constant (*T*) and the equations of motion for the string can be derived by considering the differential element shown in Figure 1(b).



(a)



(b)

Figure: (a) Diagram of a vibrating string stretched between two walls; (b) a differential element of the string of length *dx*.

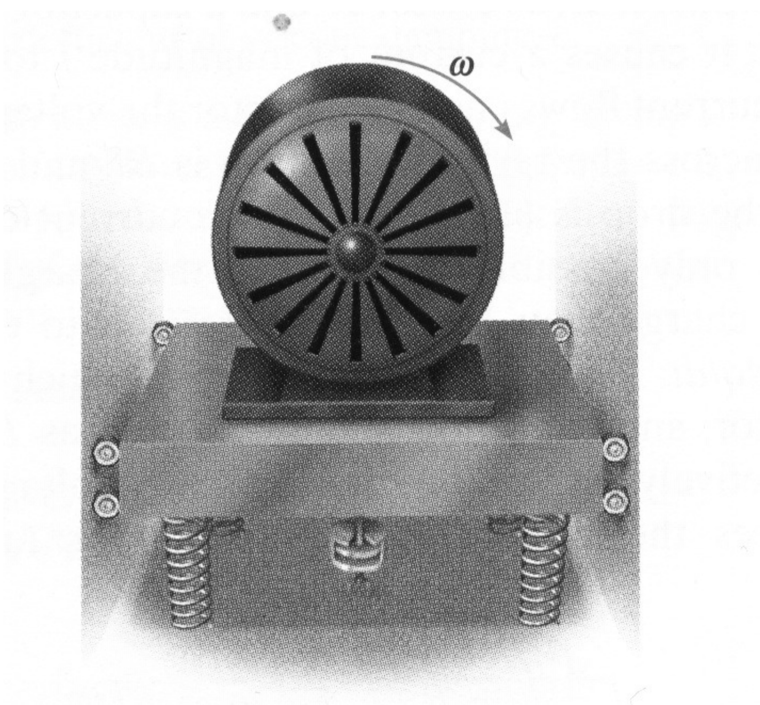
(a) Assuming small motions write down the approximate relationship between the angle of the string $\theta(x,t)$ and the displacement of the string $y(x,t)$.

(b) Using a first order Taylor series expansion with *dx* as the small variable, write down an approximate relation for $\theta(x+dx,t)$ in terms of $y(x,t)$.

(c) Assuming small motions and ignoring gravity, apply Netwon's second law ($\Sigma F=ma$) to the differential element in Figure (b) to obtain the equation of motion for the string. Hint: (1) ΣF is the sum of the forces on the element; (2) the mass of the differential element *dm* is the length *dx* multiplied by the mass per unit length γ; and (3) the acceleration is $a = \partial^2 y/\partial t^2$.

(d) Discuss in general how to solve the equation of motion for the vibrating string. What are the boundary conditions? What happens if the guitarist increases the tension in the string – show the effect mathematically and describe what happens in words?

Poblem 5: The 30 kg electric motor shown below is supported by four springs each having a stiffness of 200 N/m. If the rotor is imbalanced so that its effect is equivalent to a 4 kg mass located 60 mm from the axis of rotation. The non-dimensional damping ratio of the system is $\zeta=0.15$ ($\zeta=c/(2(km)^{1/2})$).



(a) Determine the amplitude of vibration when the rotor is turning at 95.5 revolutions per minute (RPM).

(b) A new technician arrives on Monday morning and gets a shock as he ramps the motor speed up from rest to a final speed of 150 RPM in ten seconds. What does the technician observe? Give some quantitative estimates of what ocurs and discuss the most major limitation of your analysis.