Fluid Mechanics Qualifying Examination

Sample Exam 3

Allotted Time: 3 Hours

The exam is closed book and closed notes. Students are allowed one (double-sided) formula sheet.

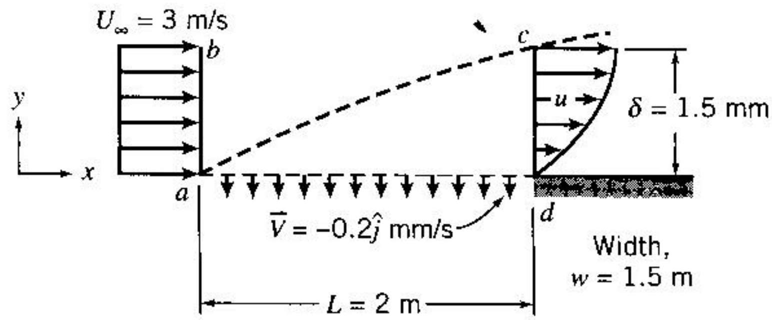
There are five questions on this exam. Answer any four (each for 25 points).

If you answer all five, the best four will be considered.

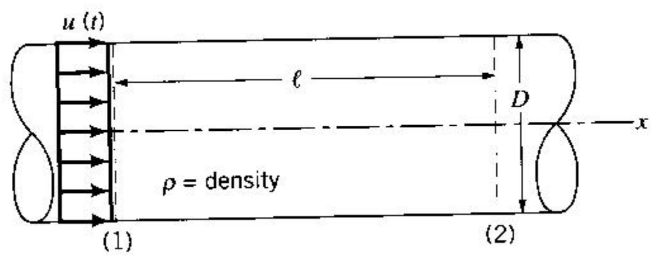
State all your assumptions, and explain your reasoning clearly.

1.

(a) Consider incompressible steady flow of standard air in a boundary layer on the length of porous surface shown. Assume the boundary layer at the downstream end of the surface has an approximately parabolic velocity profile, $\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$. Uniform suction is applied along the porous surface as shown. Calculate the volume flow rate across surface *cd*, through the porous suction surface, and across surface *bc*.



(b) Consider unsteady flow in the constant diameter, horizontal pipe shown in figure. The velocity is uniform throughout the cross-section, but is a function of time. So, V = u(t) **î**. Using control volume analysis, develop an expression for the pressure drop between (1) and (2) in terms of the instantaneous velocities u_1 and u_2 at the points 1 and 2.



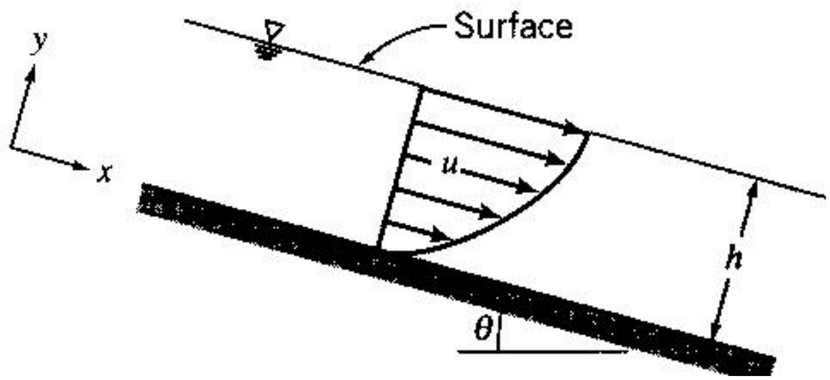
2. A liquid flows down an inclined plane (of angle θ) surface in a *steady*, *fully developed laminar* film of thickness h (see figure). The density of the liquid is ρ, the dynamic viscosity μ.

(a) State and formulate the assumptions required to solve this flow.

(b) Start with the 3D Navier-Stokes equations (mass and momentum) and simplify them to model this flow field using your assumptions.

(c) List and formulate the boundary conditions required to solve this flow.

(d) Obtain an expression for the velocity profile in the x-direction.



3. The 2D inviscid flow past a circular cylinder can be obtained by superposing a uniform flow field with a doublet. The resulting streamfunction in *r*-θ coordinates is given by

$$\Psi = Ur\left[1 - \left(\frac{a}{r}\right)^2\right]\sin\theta$$

where U is the freestream velocity of the flow, *a* is the radius of the cylinder. Assume pressure in the free-stream is atmospheric. Note that in radial coordinates, $v_r = \frac{1}{r}\frac{\partial\Psi}{\partial\theta}$, $v_\theta = -\frac{\partial\Psi}{\partial r}$.

(a) From the above expression, compute the radial and tangential components of the velocity field.

(b) What are the velocity components along the surface of the cylinder?

(c) Find an expression for the pressure distribution along the surface of the cylinder.

(d) Show that the coefficient of pressure for this flow is $C_p = \frac{p_s - p_{atmospheric}}{\frac{1}{2}\rho U^2} = 1 - 4sin^2\theta$ where p_s is the pressure at the forward stagnation point.

4. The drag force on a solid sphere (density = ρ_s) of diameter D falling at a velocity of u in a still medium (incompressible) with density ρ_∞ and viscosity μ is given by

$$F_D = 3\pi\mu Du$$

Provided $Re = \rho_\infty uD/\mu << 1$

(a) Use the above equation to determine the terminal velocity of the sphere for such low Reynolds numbers (with and without consideration of buoyancy forces).

(b) Given a transparent tube filled with an unknown fluid, a ruler, and a stopwatch, how will you estimate the viscosity of the fluid?

(c) Simplify the Navier-Stokes equations for this low Reynolds number flow.

5.

(a) Consider a compressible flow passing through a normal shock. Write down the appropriate forms of the conservation of energy, linear momentum, and mass across the shock. Express in terms of the densities, ρ_1, ρ_2, velocities u_1, u_2, pressures p_1, p_2, and enthalpies h_1, h_2 that exist on each side of the shock.

(b) A normal shock stands in a duct. The fluid is air, which may be considered an ideal gas. Properties upstream of the shock are T_1 = 5 C, p_1 = 65.0 kPa (abs), and V_1 = 668 m/s. Determine properties downstream. Sketch the process on a T – s diagram.