

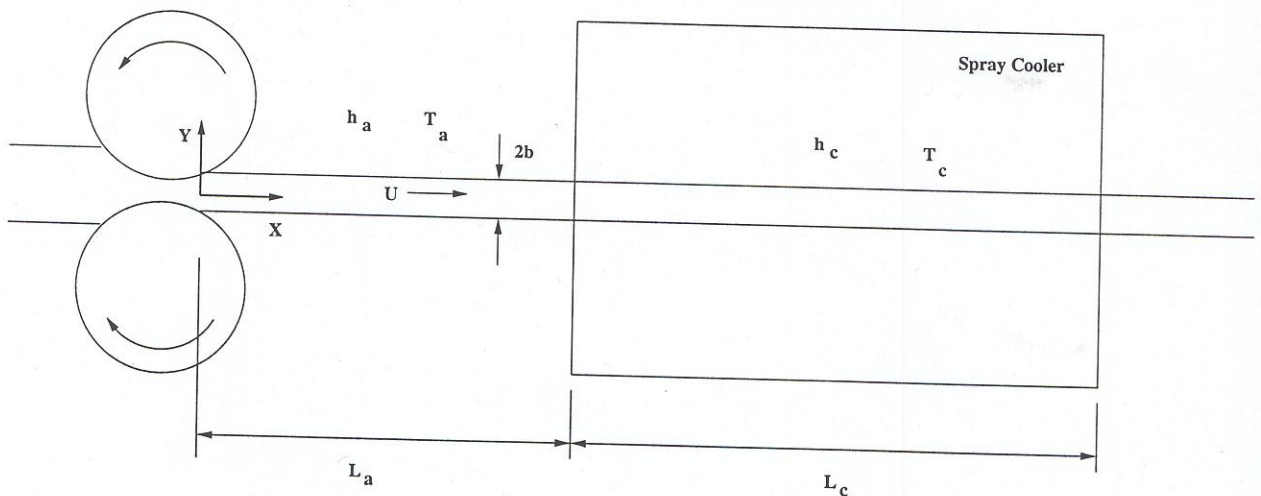
Ph.D. Qualifying Exam - Fall 1999
Fluid Mechanics and Heat Transfer Minor

1) After passing between two rolls, hot steel plate travels in room air to a spray cooler as shown. At the exit from the rolls, the plate's temperature is uniform and equal to T_h and the plate's steady speed is U . The convective heat transfer coefficient for convective cooling in air is h_a and for spray cooling is h_c . The ambient temperature of room air is T_a and for cooling water is T_c .

a) Given the dimensions and coordinates shown, and assuming steady state conditions and constant plate properties k , ρ , and c_p , derive (or write down) the equation governing heat transfer within the plate. Note, heat transfer occurs by both conduction and bulk motion of the plate.

b) Write down the boundary conditions needed to solve for the temperature distribution within the plate. Clearly indicate where each boundary condition applies. Do not solve.

c) Since the plate's temperature is unknown at the exit from the spray cooler, we have to specify a reasonable boundary condition. Estimate the orders of magnitude of $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ in the vicinity of $x = L_a + L_c$, take the ratio of these estimates, and based on this ratio, specify a reasonable boundary condition on $\frac{\partial T}{\partial x}$. Assume that $2b/L_c \ll 1$.



2) A rocket is launched from the ground with a full fuel tank. The speed of burnt fuel relative to the nozzle remains constant at V_{jet} , while the fuel's density remains fixed at ρ_f . The weight of the unfueled rocket is W_R .

a) Define a moving control volume that encloses and moves with the rocket. Starting with the integral form of the conservation of linear momentum applied to the contents of the control volume (i.e., the rocket and the unburnt fuel within the rocket), show that in the vertical direction, the momentum equation is given by

$$\frac{d}{dt}(MV) + M_R \frac{dV}{dt} = \rho_f A_{jet} V_{jet} (V_{jet} - V) - Mg - W_R - C_D \frac{1}{2} \rho_a V^2 A_R$$

where

$M = M(t)$ = mass of unburnt fuel in rocket at time t

$V = V(t)$ = instantaneous vertical velocity of the rocket (relative to the earth)

$M_R = W_R/g$ = the (fixed) mass of the rocket (without fuel)

ρ_f = fluid density

A_{jet} = the cross-sectional area of the rocket's nozzle

g = gravitational acceleration

W_R = weight of the rocket

C_d = the drag coefficient for air flow past the rocket

ρ_a = density of air

A_R is the rocket's projected area in the direction of motion

It is assumed that the flow of burnt fuel out of the nozzle is uniform across the cross-sectional area A_{jet} and that the velocity of the fuel within the rocket is equal to the rocket's instantaneous velocity. Note that the total instantaneous drag D on the rocket is given by $D = C_d \frac{1}{2} \rho_a V^2 A_R$. In addition, recall that the surface integral giving the flux of momentum out of/into a moving control volume is given by

$$\int_S \rho \mathbf{u} (\mathbf{w} - \mathbf{u}) \cdot \hat{\mathbf{n}} dS$$

where S is the surface enclosing the control volume, \mathbf{u} is the fluid velocity across S (relative to a fixed reference frame), \mathbf{w} is the velocity of the surface S (again relative to a fixed reference frame), and $\hat{\mathbf{n}}$ is the outward unit normal to S .

b) Use the same moving control volume and show that the integral form of the conservation of mass leads to the following equation:

$$\frac{d}{dt}(M) = -\rho_f A_{jet} V_{jet}$$

