

1. A guitar string of length L and with an initial tension T is plucked at the mid-point and released. The resulting vibrational motion is given by

$$u(x, t) = \frac{8a}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \sin \frac{(2n+1)\pi x}{L} \cos \frac{(2n+1)\pi t}{L}.$$

Recall from the class notes that the initial condition on $u(x, t)$ is

$$u(x, 0) = \frac{2a}{L} \begin{cases} x, & \text{for } 0 \leq x \leq \frac{L}{2}, \\ L - x, & \text{for } \frac{L}{2} \leq x \leq L. \end{cases}$$

- With the help of matlab, plot the solution $u(x, 0)$ by considering the first m terms of the Fourier series with $m = 1, 2, 5, 10, \text{ and } 50$ respectively. You will find the code `guitar_string.m`, posted on the class website, useful for this purpose.
- Show that each term in the Fourier series satisfies the boundary conditions $u(0, t) = 0$ and $u(L, t) = 0$.
- Show that the initial condition is not satisfied by any of the single Fourier terms.