1. Problem 14.2 from the text by the separation of the variables technique.

2. Consider the following differential equation that is used to estimate the growth of a population.

$$\frac{dP}{dt} = (\alpha - \beta P)P.$$

Here, $P(t)$ is the population size, α and β are some constants and the quantity $(\alpha - \beta P)$ is the growth rate of the population. Thus, if this quantity is less than zero, the population size decreases and if it is greater than zero, the population size increases.

(a) Suppose that $P(0) = 0$. By observation of the differential equation, can you identify the solution?

(b) Suppose that $P(0) = \frac{\alpha}{\beta}$. Again, by observation of the differential equation, can you identify the solution?

(c) As $t \to \infty$, suppose that the population reaches a steady-state. What are the possible values for the population size?

(d) Suppose $P(0) = P_0$. Using the separation of variables method, obtain the solution to this equation. Draw the solution curve when $P_0 < \frac{\alpha}{\beta}$ and when $P_0 > \frac{\alpha}{\beta}$.

3. Consider the problem of draining of a tank discussed in the class. Derive the governing differential equation. Using the separation of variables method, obtain the solution to $h(t)$ which is the height of the liquid level in the tank at any time t. Using this expression, derive the time it takes to drain the tank. Plot the curve $h(t)$ against t for $h(0) = 1$ m and $d_t = 0.5$ m and $d_o = 0.05$ m, 0.1 m and 0.2 m. Show all the curves in the same plot window. You can use `Matlab` for this purpose.