1. The Wronskian $W(f_1, f_2, \ldots, f_n)$ of a set of functions $f_i(x), i = 1, 2, \ldots, n$ is defined by

$$W(f_1, f_2, \ldots, f_n) = \begin{vmatrix} f_1 & f_2 & \cdots & f_n \\ f' & f_2' & \cdots & f_n' \\ \vdots & \vdots & \vdots & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & \cdots & f_n^{(n-1)} \end{vmatrix}$$

   Suppose that the set of solutions $y_1, y_2, y_3, \ldots, y_n$ constitute are n solutions for an nth-order `normal` linear ordinary homogeneous differential equation. Then, it can be shown that these solutions are linearly independent if and only if the Wronskian of these solutions is non-zero on the interval (i.e., for all x on I) over which the solutions are valid. Such a linearly independent set is called the fundamental set of solutions or the basis of the differential equation.

   Consider the differential equation $y'' - 9y = 0$ on $I = (-\infty, \infty)$. Show that e^{3x}, e^{-3x} and $5\sinh 3x - 2e^{-3x}$ satisfy this equation. Calculate the Wronskian W of these solutions and show that they are not linearly independent. Next, calculate the Wronskian of any two and show that they are linearly independent.

2. Use the variation of parameters method to find a complete solution to $2xy'' + y' = 10x^{\frac{1}{4}}$ given that $y_1(x) = 1$ and $y_2(x) = x^{\frac{1}{2}}$.

3. Find a complete solution of the differential equation given that $y'' + 2y' + y = 0$ with $y_1 = e^{-x}$. Use the method of reduction of order for this problem.

4. Find complete solutions of the following two equations:
   (a) $y'' - 3y' + 2y = 0$ and
   (b) $y'' - 3y' + 3y = 0$.

5. Obtain an expression for the radial displacement in a cylindrical pressure vessel subjected to internal and external pressures (as discussed in the class). Note that a general solution was obtained in the class. You will need to apply the boundary conditions to determine the constants of integration. Then, assuming that the external pressure is zero and the thickness of the cylinder is t, show that the expression for hoop stress reduces to that derived in elementary mechanics of materials courses. You can use `Maple` for this exercise. Also, plot the variation of S_{rr} and $S_{\theta\theta}$ with the radius for the following sets of data:

   - $p_i = 20$ MPa, $p_o = 0$ MPa, $a = 0.1$ m and $b = 0.15$ m.
   - $p_i = 20$ MPa, $p_o = 0$ MPa, $a = 0.1$ m and $b = 0.11$ m.
   - $p_i = 20$ MPa, $p_o = 10$ MPa, $a = 0.1$ m and $b = 0.15$ m.
   - $p_i = 20$ MPa, $p_o = 10$ MPa, $a = 0.1$ m and $b = 0.11$ m.