Fluid Mechanics Qualifying Examination

Spring 2024

Allotted Time: 3 Hours

- The exam is closed book and closed notes. Students are allowed bring a one-page (double-sided) hand-written formula sheet.
- There are **FIVE** questions on this exam. Answer any **FOUR** (each for 25 points).
- If you answer all questions, the best FOUR will be considered.
- State all your assumptions, and explain your reasoning clearly. Where applicable, give detailed explanations of flow phenomena
- Use either a symbol or a logical value for any missing quantity. You must explain clearly the rationales for your choice.
- Please number your pages.

1. (a) The x and y components of a velocity field are given by $u = x^2y$ and $v = -xy^2$. Determine the equation for the streamlines of this flow. Please sketch this family of streamline. **(10 points)**

   (b) After a rainfall, the sediment concentration at a certain point in a river increases at the rate of 100 parts per million (ppm) per hour. In addition, the sediment concentration increases with distance downstream as a result of influx from tributary streams. This rate of increase is 50 ppm per mile. The stream flows at 0.5 mph. A boat is used to survey the sediment concentration. The operator is amazed to find three different apparent rates of change of sediment concentration when the boat travels upstream, drifts with the current, or travels downstream. Explain physically why the different rates are observed. If the speed of the boat is 2.5 mph, compute the three rates of change. **(15 points)**

2. (a) When the drain plug is pulled, water flows out from the bottom of a large, cylindrical open bath tub. Assume viscous effects are negligible, and flow is steady. Estimate how much longer it takes to empty the entire bathtub, than just the first half. Provide an explanation for your result. Make any other assumptions necessary to solve this problem. **(15 points)**

   (b) Consider the water jet falling from a kitchen faucet of diameter D. The velocity is low enough that the jet is laminar and smooth, and does not atomize into droplets at any point. Find an expression for the diameter of the jet as a function of the vertical height $y = -h$ to which the jet has fallen ($y = 0$ is the exit plane of the jet). **(10 points)**

3. (a) Imagine you are accelerating your car from 0 to 60 mph. Describe in detail the changes in the total drag forces experienced by the vehicle through the acceleration, and what causes them. **(10 points)**

   (b) Which would drop to the ground faster: a flat sheet of paper held parallel to the ground and released, or if the paper were wadded into a ball. Provide an explanation for your answer. **(5 points)**

   (c) A round thin disk of radius R is oriented perpendicular to a fluid stream. The pressure distributions on the front and back surfaces are measured and presented in the form of the following pressure coefficients: $C_{p,f} = 1 - \left(\frac{r}{R}\right)^6$, $C_{p,r} = -0.42$. Calculate the drag coefficient of the disk. **(10 points)**

4. (a) Consider steady, fully developed laminar flow of a viscous liquid down an inclined surface. The liquid layer is of constant thickness h. Use a suitably chosen differential control volume to obtain the velocity profile. Develop an expression for the volume flow rate per unit width. **(10 points)**

(b) Consider the viscous flow over a horizontal flat plate, forming a boundary layer. Starting from the leading edge of the plate at $x = 0$, describe how the boundary layer develops, and how does it transition from laminar flow to turbulence? **(10 points)**

(c) NASA recently demonstrated flight of its Ingenuity drone helicopter on Mars. On Mars, gravity is 3.71 $^{m}/_{s^2}$ and atmospheric gas density is ~ 0.02 $^{kg}/_{m^3}$. If you were to design a helicopter with a specified weight for Martian flight, what are the aerodynamic issues you will consider, and how will they differ from flight on earth? How will you approach this problem? **(5 points)**

5. (a) A blastwave propagates outward from an explosion. At large radius, curvature is small and the wave may be treated as a strong normal shock (the pressure and temperature rise associated with the blast wave decrease as the wave travels outward). At one instant, the blast wave front travels at M_i with respect to the undisturbed air. Lay out the steps to determine the speed of the air behind the blastwave (a) relative to the wave, and (b) as seen by an observer on the ground. Use standard symbols for the different quantities. **(15 points)**

   (b) A converging-diverging nozzle is attached to a very large tank of air in which the pressure is p_{tank} and the temperature is T_{tank}. The nozzle exhausts to the atmosphere where the pressure is p_0. If the exit area of the nozzle is given by A_n, how would you determine the flow rate through the nozzle. Assume flow is isentropic. Use standard symbols for the different quantities. **(10 points)**