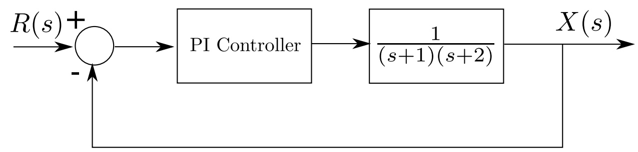
PhD Qualifier: Control Spring 2024

The duration of this exam is 3 hours. There are 5 questions and the students must work 4 of them. If more than 4 questions are attempted, the student must specify which 4 are be graded. If not specified, the first 4 questions will be graded. Each question is 25 points and you need 70 points to pass this exam. Good Luck!

1	2	3	4	5

Problem 1 Consider the following system: the controller block is a PI controller with proportional gain K_P and integral gain K_I. The reference signal input is $R(s)$ and the system state output is $X(s)$.



(a) Derive the closed-loop transfer function

$$G_{\text{CL}}(s) = \frac{X(s)}{R(s)}$$

(b) Show that a sufficient condition for stability is that the PI controller gains satisfy

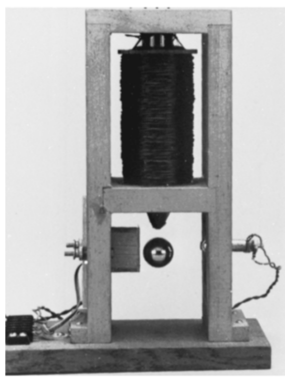
$$K_I > 0 \text{ and } K_P > \frac{1}{3}K_I - 2$$

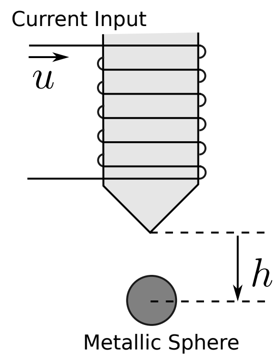
(c) Suppose that a unit step input $R(s)$ is applied. What value does the state $x(t)$ approach as $t \to \infty$? Justify your answer.

Problem2. A magnetic levitation device shown below is modeled by the differential equation

$$m\frac{d^2h}{dt^2} = mg - k\frac{u^2}{h^2}$$

where m is the mass of a metallic sphere, g is the gravitational acceleration constant, k is a positive constant, h is the distance between the electromagnet and the metallic sphere, and u is the electromagnet's current input.





(a) Convert the ODE above into a system of first-order vector differential equations of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

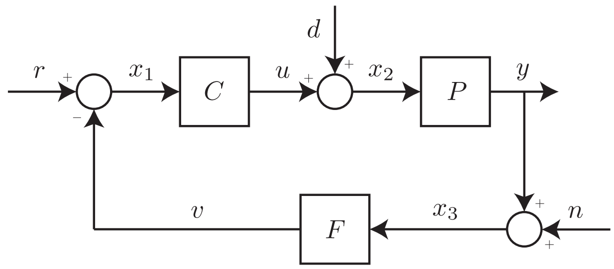
(b) Determine the height h_0 at equilibrium as a function of a steady equilibrium current input u_0 and the other constants of the problem.

(c) Linearize the system around the equilibrium point from part (b) to arrive at a system model of the form:

$$\delta\dot{\boldsymbol{x}} = \boldsymbol{A}\delta\boldsymbol{x} + \boldsymbol{B}\delta u$$

where $\delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_0$ and $\delta u = u - u_0$ are the deviations of the state and control, respectively, from their equilibrium.

Problem 3. Here's a block diagram for a generic single-input, single-output, linear time-invariant feedback control system...



... labeled thus:

r	reference input	$x_1 = r - v$
u	controller output	$x_2 = u + d$
d	external disturbance	$x_3 = y + n$
y	plant output	
n	sensor noise	
v	sensor output	
$C(s)$	transfer function of controller	
$P(s)$	transfer function of plant	
$F(s)$	transfer function of sensor	

Your responses to the following three questions will be weighted equally in grading this part of the exam.

1. The system above is *well posed* if and only if the internal signals x_1, x_2, and x_3 are determined uniquely by the three inputs r, d, and n. Give an example of three nonzero transfer functions $C(s)$, $P(s)$, and $F(s)$ for which the system is not well posed.

2. For a system like the one shown, what's the difference between input-output stability and internal stability? Give an example of three nonzero transfer functions $C(s)$, $P(s)$, and $F(s)$ for which the system is input-output stable but not internally stable.

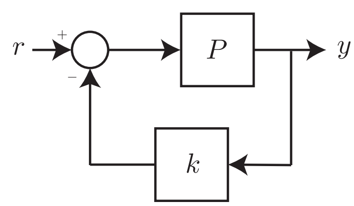
3. Suppose that

$$C(s) = 1, \qquad P(s) = \frac{1}{s(s+b)}, \qquad F(s) = 1$$

for some positive real constant b. This system is internally stable. Compute the system's gain margin.

Problem 4. Your responses to the following four questions will be weighted equally in grading this part of the exam.

1. What does it mean for a linear time-invariant system to be *minimum phase*?
2. Provide examples of transfer functions $P(s)$ for two systems, one minimum phase and the other not, that exhibit the same Bode magnitude plot and the same steady-state step response.
3. Qualitatively, how would your two systems' step responses differ prior to reaching a steady state?
4. Suppose that each of your two transfer functions were enclosed in a simple feedback loop like this



in which k is a positive constant gain. Find a value of k for which the closed-loop system with transfer function $Y(s)/R(s)$ is stable for your minimum-phase $P(s)$ but not for your other $P(s)$.

Problem 5.

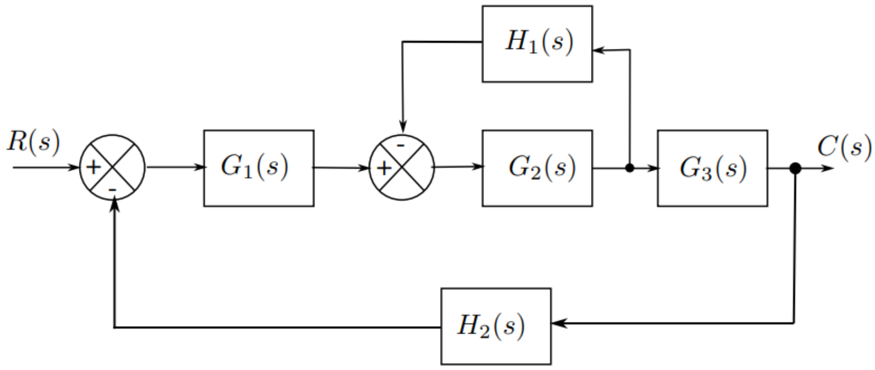


Fig. 1: Closed loop control system

1. Determine the transfer function for the system block diagram shown in figure 1.

   For this system

$$G_2(s) = G_3(s) = \frac{1}{1 + s\tau}, \quad H_1(s) = K_1, \quad H_2(s) = K_2. \tag{1}$$

   comment on the stability of this system (i.e. stable, unstable, or conditionally stable) when

   (a) $G_1(s)$ is a constant K_P,

   (b) $G_1(s)$ is an integrator of the form $\frac{K_I}{s}$

Short table of Laplace transforms	
$f(t) = \mathcal{L}^{-1}\mathcal{F}(s)$ for $t > 0$	$\mathcal{L}(f(t)) = \mathcal{F}(s)$
Unit impulse	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
t^N	$\frac{N!}{s^{N+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-\alpha t}\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
$\dot{f}(t)$	$s\mathcal{F}(s) - f(0)$